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An equilibrium model to predict shoreline rotation of pocket beaches

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ABSTRACT

A novel and simple beach-evolution model for estimating shoreline rotation at sandy pocket beaches is presented. The model is based on the assumption that the instantaneous changes to the planview shape of the shoreline depend on the long-term equilibrium planview shape. Two years of shoreline observations extracted from video images of three artificially embayed beaches of Barcelona and hourly wave timeseries are used to validate the model. Numerical model results and field observations show an excellent agreement with an RMSE less than 1.5 m. The model successfully reproduced the shoreline response over a range of scales (months and years). Because of its simplicity and its computational efficiency, the model provides a powerful tool to understand the dynamics regulating the evolution of pocket beaches and predict temporal patterns in beach rotation.

1. Introduction

Changes in the shoreline location of sandy beaches are the result of a large number of processes and mechanisms which interact on a variety of spatial and temporal scales (e.g., De Vriend et al., 1993a, 1993b). Beaches are in fact complex dynamic systems and respond to waves and currents through a series of changes that can occur at different time scales (Werner, 2003). In the last decade, several approaches have been developed for predicting beach changes induced by wave action. The approaches can be broadly divided into two categories: data-driven and process-based. The term data-driven refers to models that entirely rely on the presence of a pre-existing data-set to develop what is usually a site-specific predictor. An example of a simple data-driven model is a regression analysis relating changes in shoreline position to some averaged measure of the previous offshore wave climate. Analyses of this type have been presented by several authors also for the study of beach rotation on embayed (e.g., Harley et al., 2010) or pocket beaches (e.g., Ojeda and Guillén, 2008; Turki et al., 2013). Other more complicated data-driven predictors can also be developed (e.g., artificial neural networks) but the theoretical approach remains the same. Overall, these models tend to be site-specific and generally require an extensive data set for calibration purposes. At the opposite end of the spectrum one can find the so-called process-based models which entirely rely on the presence of a set of equations to address the balance between the driving forces and the shoreline response. Also in this case a variety of models with different degree of complexity have been proposed: from models addressing only the behavior of the shoreline under the presence of a longshore current (e.g., GENESIS, Hanson, 1989) to models that address as many processes and interactions as computationally feasible (e.g., DELFT3D, Roelvink and Van Banning, 1994). In this context, it is worth pointing out that a model built including more processes does not necessarily results in more precise predictions. In fact, at present, from the standpoint of their practical application to coastal management, such complex models may be still considered to be at a relatively early stage of development and require tuning of calibration coefficients (e.g., friction coefficient) or present approximate description of processes (e.g., use of a single grain size). For this reason, highly simplified models, often termed as "heuristic", have also been proposed. The term heuristic is usually associated to the development of models that look at the system's behavior over long temporal scales and assume that under steady wave forcing there is an equilibrium configuration. For example, the assumption of equilibrium shapes for the cross-shore beach profile (e.g., Dean, 1977; Larson and Kraus, 1989; Plant et al., 1999) remains a powerful tool to study the effect of engineering schemes (e.g., Dean, 1991) or even the effect of climate change on shoreline erosion (e.g., Bruun, 1954).

For the specific case of predicting planform shoreline changes, a semi-heuristic/conceptual approach has also been proposed. Wright et al. (1985) developed a model to study beach morphology using the Dean parameter $\Omega = H_b / (w_s T)$ (Gourlay, 1968; Dean, 1973) which is a function of the breaking wave height, H_b, the sediment fall velocity,

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w_s, and the wave period, T. Wright et al. (1985) studied shoreline changes under the hypothesis that its instantaneous response depends on the instantaneous "disequilibrium" which is the difference between the instantaneous and the equilibrium wave energy. They found that the equilibrium beach state is related to the long-term wave climate and that the shoreline response is not particularly well correlated to the instantaneous energy conditions. The "equilibrium" approach has also been pursued by other authors using numerical simulations (e.g., Kriebel and Dean, 1985; Larson and Kraus, 1989). Observations of physical beach processes suggested that shoreline response to steady-wave conditions is approximately exponential in time and may be approximated using:

$$y(t) = y_{eq} \cdot \left(1 - e^{-\alpha \cdot t}\right) \tag{1}$$

where $y(t)$ is the shoreline position at time t, y_{eq} is the equilibrium shoreline position determined by the forcing at time t, and α is a constant governing the rate at which the shoreline approaches equilibrium. The response suggested by Eq. (1) has been observed in small- (Swart, 1974) and large-scale (e.g., Sunamura and Maruyama, 1987) laboratory experiments as reported by Larson and Kraus (1989). The differential equation governing the exponential beach response to steady-wave conditions is given as:

$$\frac{dy(t)}{dt} = \alpha \cdot \left(y_{eq} - y(t)\right) \tag{2}$$

This relation was also used by Miller and Dean (2004) to develop and calibrate a simple model which relates shoreline change to its disequilibrium position using a number of hydrodynamic (e.g., wave conditions, and tides) and morphological (e.g., berm height) parameters. More recently, Yates et al. (2009) used a 5-year dataset of shoreline location and wave conditions to develop a simple equilibrium shoreline model able to reproduce the movement of the cross-shore beach profile. This model assumes that the shoreline response to the wave energy is not sensitive to wave direction and does not take into account the water level. Overall, Eq. (2) has proved useful to study physical processes related to shoreline variability and has been successfully utilized to model shoreline changes associated with cross-shore processes (Davidson and Turner, 2008; Davidson et al., 2011).

The focus of the present work is the development of a robust model based on Eq. (2) and capable of forecasting shoreline changes over long timescales and with a quantifiable degree of accuracy. Two years of observations of shoreline position and wave timeseries at Barcelona beaches, described in Section 2, are used to validate the beach-evolution-model developed in Section 3. Results are presented in Section 4 while the performance of the model and its applicability are discussed in Section 5.

2. Field observations

Three artificial embayed beaches of Barcelona, on the north-eastern coast of Spain (NW Mediterranean) (Fig. 1a) were studied in the present work. These sandy beaches are affected by the same wave conditions but have different morphological characteristics. Bogatell and Nova Icaria are characterized by a coarser mean grain size (0.75 mm) while the beach length is 600 m at Bogatell and 400 m at Nova Icaria. The length of the third beach studied, Somorrostro, is the same as Nova Icaria though the sediment is finer (mean grain size of 0.45 mm).

2.1. Wave data

Hourly wave data were obtained from a hindcast analysis for the period between 1991 and 2008 (Reguero et al., 2012). The hindcast wave database has a temporal resolution of 1 h and provides spectral sea state parameters in deep water including significant wave height, H_s, mean period, T_m, peak period, T_p, and mean direction with respect to the North, Dir. Hindcasts were calibrated with instrument data and were propagated to the breaking using the SWAN model. A detailed description of this analysis can be seen in Reguero et al. (2012). Once propagated, the wave height, H_b, and the wave direction, Dir_b, at breaking were determined. Timeseries of H_b and Dir_b between 2005 and 2007 are plotted in Fig. 2a.

2.2. Video data

Daily mapping of the shoreline position at the three embayed beaches of Barcelona was performed by Turki et al. (2013) from March 2005 to March 2007 using a video system (Holman and Stanley, 2007) installed on October 2001 in a nearby building at a height of 142 m (Fig. 1a). The system is composed of five cameras but in this study only camera C_1 and C_5 are considered. Camera C_5 covers the Bogatell and Nova Icaria beaches while Somorrostro is captured by camera C_1. Images were provided by the Coastal Ocean Observatory at the Instituto de Ciencias del Mar-Consejo Superior de Investigaciones Cinetificas (ICM-CSIC) in Barcelona (Spain). Shorelines, all related to the same tide level (0.2 m), were extracted from the time-exposure video images, and the shoreline position was measured at a series of cross-shore profiles (from P_1 to P_{10}, see Fig. 1b). Results of video-derived shorelines were smoothed using a cubic interpolation and compared favorably (differences less than 1.2 m) with measurements of shoreline positions obtained through differential Global Positioning System survey (Turki et al., 2013).

2.3. Shoreline rotation

Shoreline rotation was studied at Barcelona beaches during a period of two years (March 2005–March 2007) when human activities (beach nourishments and sand redistribution along the beach after storms) were carried out. Shoreline rotation was evaluated along a series of cross-shore profiles (from P_1 to P_{10}) spaced in time between 1 to 4 days depending on the availability of the video images (an example of the resulting timeseries is illustrated in Fig. 2b where observations from Nova Icaria beach are presented). Under the assumption of linear shape of the shoreline and a constant cross-shore profile, Turki et al. (2013) used the shoreline data to develop a simplified model which separates the overall shoreline movement into the contributions of rotation and translation.

3. Model development

3.1. Basic assumptions

According to the overview presented in the introduction, natural variability in shoreline position could be studied using a simple equilibrium approach as initially proposed by Wright et al. (1985) and then further explored by other authors (e.g., Miller and Dean, 2004; Yates et al., 2009). This approach is pursued in this research and has been applied to describe changes in the plan-form rotation of pocket beaches. An approach based on plan-form equilibrium implies a series of hypotheses which will be described in detail in the remaining of this section.

The total shoreline movement is a combination of a cross-shore profile translation and a plan-form rotation. We assume that the plan-form rotation is essentially independent of the cross-shore beach translation and is produced around a pivotal point acting as a central axis of the beach (hypothesis 1). The pivotal point is generally located in the center region of the beach, as shown Fig. 3, where the beach rotates from its initial position (dashed-black line) to a new position (solid-black line) under steady conditions.

Furthermore, hypothesis 2, the cross-shore beach profile has an equilibrium form and maintains its shape along the coast at all times, including when extreme changes are produced by storms. This approach

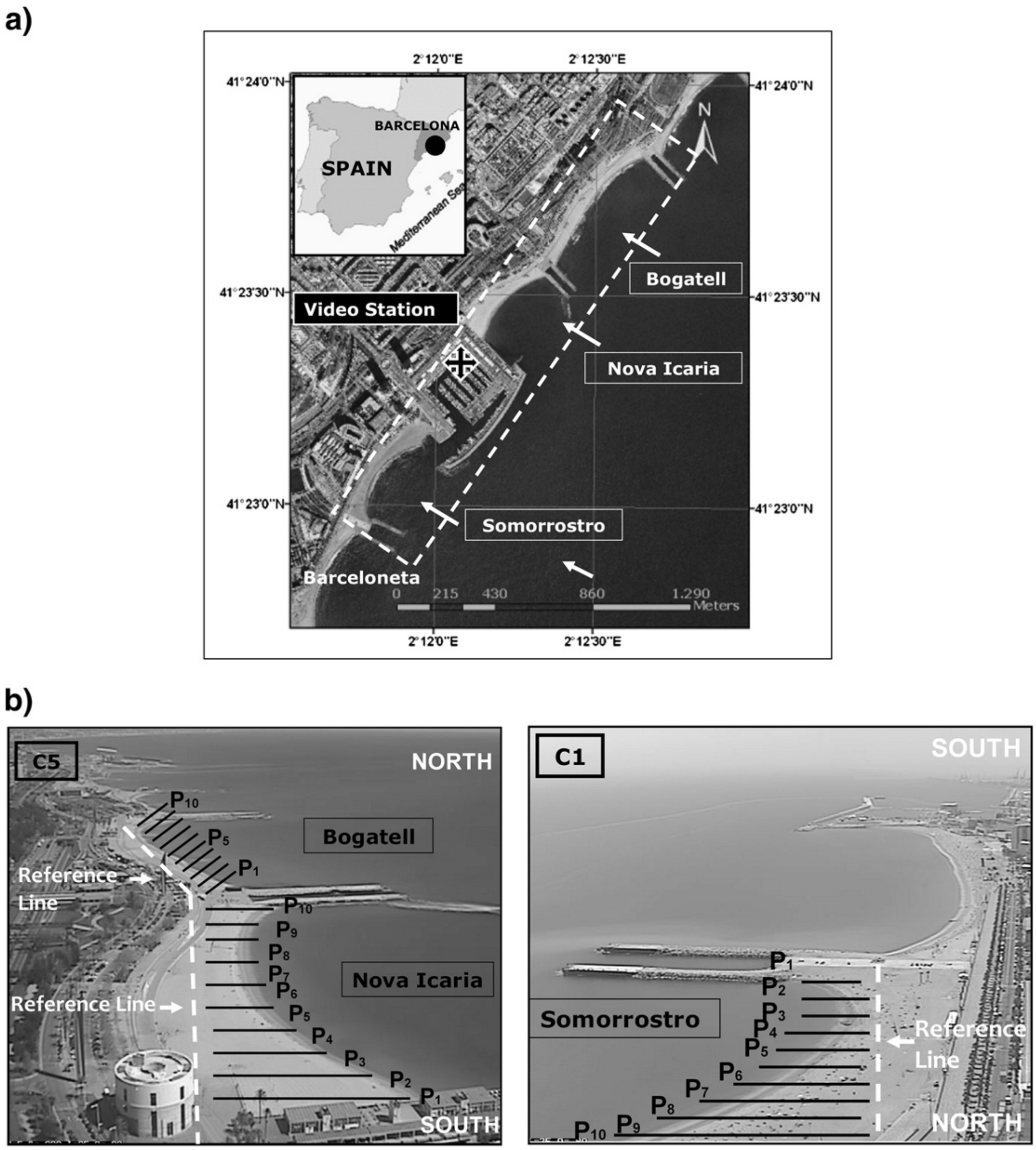

Fig. 1. (a) Study area; Barcelona City beaches. (b) Oblique video images: Camera 1 (C_1) shows Somorrostro beach. Camera 5 (C_5) shows Bogatell and Nova Icaria beaches. The white dashed line shows the reference line for the calculation of the shoreline position while the black lines indicate the position of the cross-shore profiles used to analyze beach rotation.

has been used in most studies dealing with beach plan-form and originated with the mathematical study of Pernard-Considère (1954) to analyze shoreline response to wave action.

The 3rd hypothesis is simply related to the lack of sources or sinks affecting the overall sediment balance of the beach. This is a typical hypothesis in morphodynamic studies and essentially implies that sediments are conserved and characterized by their same mean size (which is also assumed to not change over time).

The parabolic configuration of the beach plan-form can be simplified into a linear shape (Fig. 4a). Under steady wave conditions, the initial shoreline (dashed-black line) moves to a new position which implies, for example, an advance (R) of the right section of the beach or a retreat ($-R$) of the left one. In terms of sediment transport, an alongshore movement of sand from the updrift end to the downdrift one (black arrows in the figure) is produced generating erosion at one end of the beach ($-R$ and $-V$) and accretion ($+R$ and $+V$) at the opposite end. If the cross-shore profile shape does not change and the shoreline is linear, any position of the shoreline is sufficient to describe changes resulting from steady wave action. We hypothesize that sand is redistributed along the beach at a constant rate (hypothesis 4) and that when the wave climate changes, the beach responds uniformly by altering its orientation to respond to the new wave conditions.

Finally, hypothesis 5, the alongshore sediment transport can be expressed in terms of breaking wave quantities (wave height and angle between the shoreline position and the wave crest) and the alongshore gradient in wave height is null or negligible. This implies that nearshore currents associated to gradients in wave height (e.g., the cell circulation associated to the presence of rip currents) are not simulated and their effect on morphological change is disregarded.

3.2. Governing equations and parameter description

Under the hypotheses previously introduced, the rate of shoreline response $\frac{dR}{dt}$ can be described as proportional to the difference between the instantaneous position, $R(t)$, and its long-term equilibrium one, R_∞:

$$\frac{dR(t)}{dt} = \omega \cdot (R_\infty - R(t)) \tag{3}$$

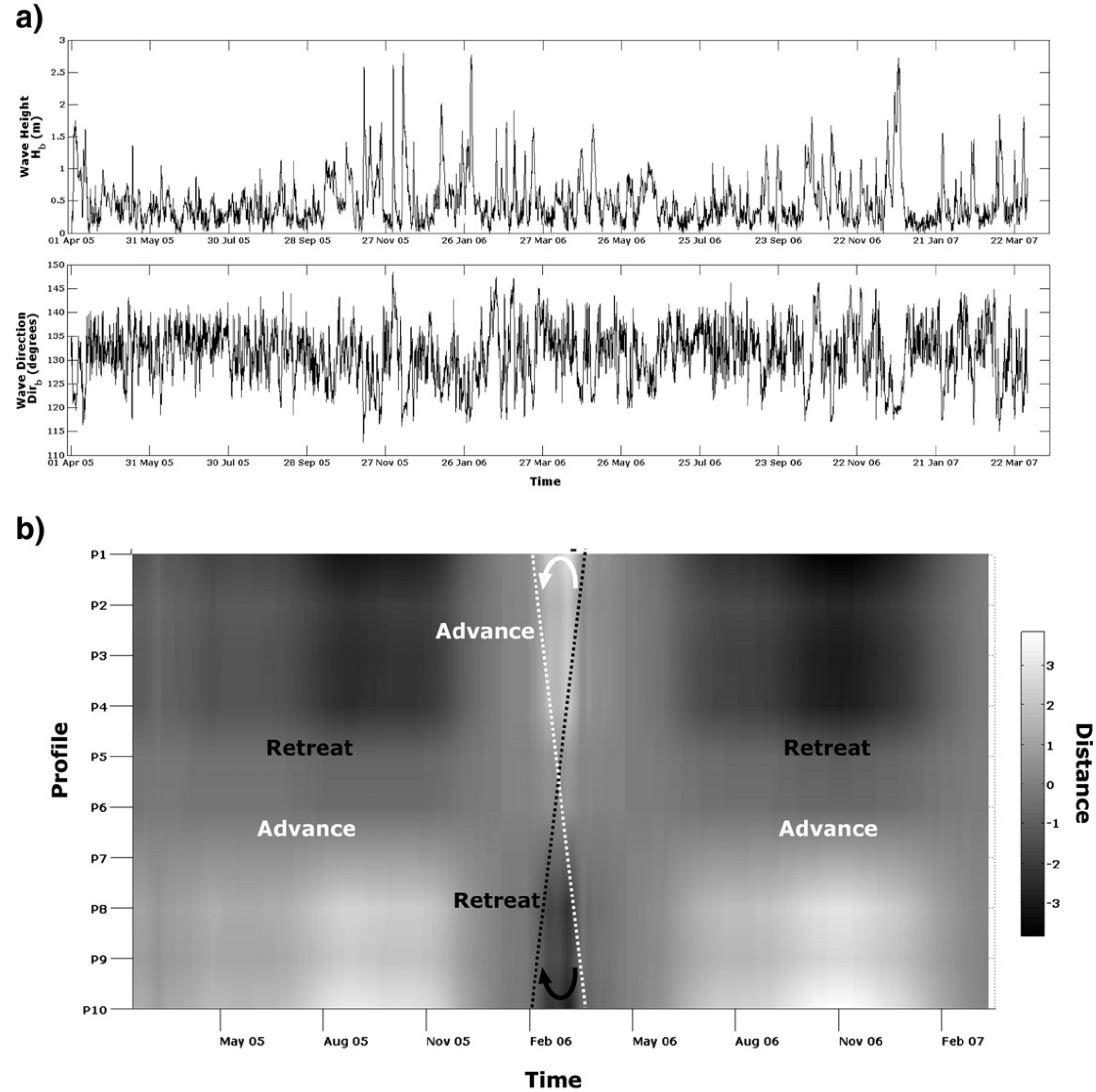

Fig. 2. (a) Timeseries of wave height (H_b) and direction (Dir_b) at Barcelona beaches extracted from hindcast data between 2005 and 2007. (b) Two-year shoreline rotation at Somorrostro beach. Rotation is maximum near the edge of the beach (P_1 and P_{10}) and minimum (or zero) at the central sections (P_5 and P_6) which approximately corresponds to the beach pivotal point. An example of a rotation event is shown in February 2006 (the black and white dotted lines help visualizing the counter-clockwise shoreline rotation).

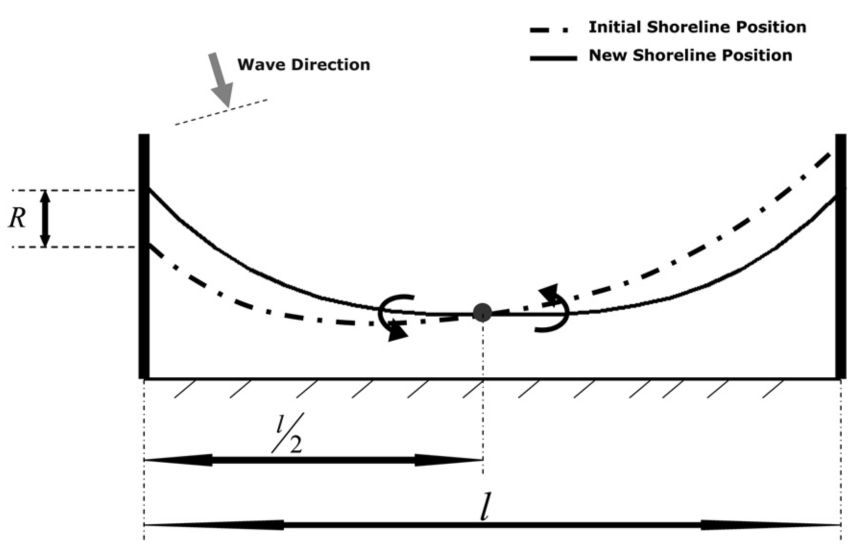

Fig. 3. Sketch of shoreline rotation. The initial position (dashed-black line) rotates to the new one (solid-black line) under steady wave conditions resulting in an overall shoreline movement R at the edge of the domain.

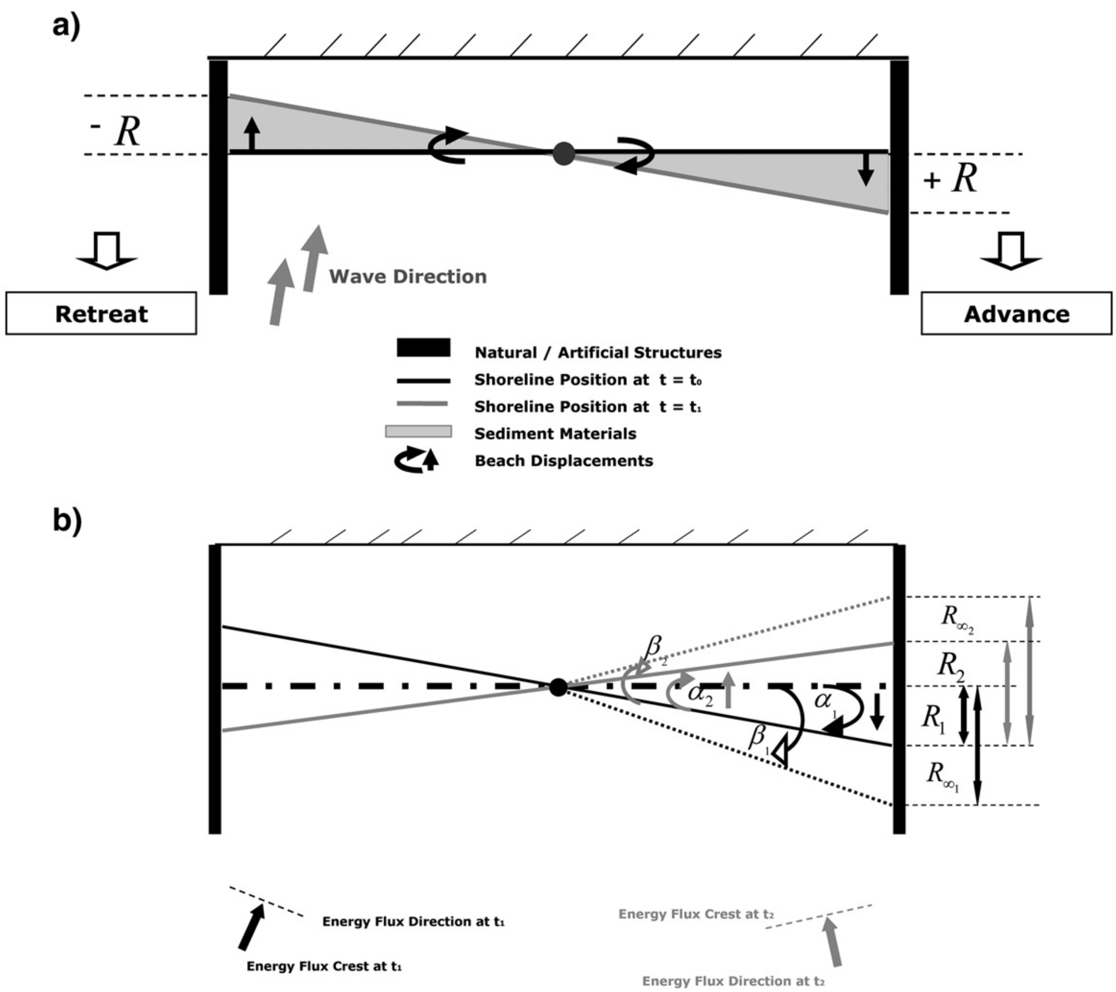

Fig. 4. (a) Simplified model of shoreline rotation. The beach plan-form is assumed to be linear and rotates around a pivotal point (black circle) from an initial position (dashed–dotted black line) to a new one (solid black line) generating a retreat on one side ($-R$) and an advance on the other ($+R$). This corresponds to a loss ($-V$) and gain ($+V$) of sediment at both sides of the beach. (b) Synopsis of shoreline rotation under two different steady wave conditions produced successively at t_1 and t_2. The theoretical equilibrium configurations (constant wave forcing) would be related to angles β_1 and β_2 while the angle between the shoreline and the wave crests at the end of t1 and t2 are α_1 and α_2, respectively. The theoretical equilibrium configurations $R_{\infty1}$ and $R_{\infty2}$ and the values associated to the finite duration of the wave climate, R_1 and R_2, are also displayed.

where ω is the rate of beach change. A simplified process of beach rotation is shown in Fig. 3b. From the initial beach plan-form (dashed-black line), a rotation can be produced assuming steady wave conditions. The beach plan-form could not reach its equilibrium response R_∞ instantaneously and its rotation is limited to a response R. The rationale behind Eq. (3) is that the rate of shoreline response $\left(\frac{dR(t)}{dt}\right)$ is determined by two parameters: (1) the equilibrium shoreline response R_∞ and, (2) the beach change rate ω which is inversely proportional to the characteristic time scale, T_s, of the shoreline response $\left(\omega = \frac{1}{T_s}\right)$ T_s governs the time required for the shoreline to respond to new forcing conditions and to reach a new equilibrium position. For a constant value of T_s, Eq. (3) can be solved analytically and has the following exponential form (Kriebel, 1986):

$$R(t) = R_\infty \cdot \left(1 - e^{-\frac{1}{T_s} \cdot t}\right). \tag{4}$$

The shoreline response R reaches 64% of its equilibrium value after a period of time equivalent to T_s while over 99% of the equilibrium response would be achieved after a time equal to $5T_s$ (Kriebel, 1986). T_s decreases with increasing energy conditions and is directly proportional to the physical characteristics of the beach, such as the beach length and the sediment grain size. In reality, because of the changing wave conditions, the characteristic time scale T_s changes over the time.

3.2.1. Equilibrium shoreline response (R_∞)

The equilibrium shoreline response can be reached if the initial position of the beach were allowed to respond instantaneously to wave forcing or if wave conditions did not change for duration equal or longer than $5T_s$. The equilibrium response R_∞ can be thought as the potential rotation of the shoreline to become parallel to the angle of wave crest at breaking, β. However, in the model wave conditions change every hour leading to a rotation of the initial plan-form with an angle α smaller than β. These parameters are described in Fig. 4b where the beach is subjected to two different wave fields each characterized by a different direction. Under the first wave field, an equilibrium shoreline response $R_{\infty1}$ (dotted black line) could potentially be achieved. The equilibrium shoreline position would be characterized by an angle β_1 (corresponding to the angle between the initial shoreline position, dashed–dotted black line, and the wave crest associated to t_1). In reality, in one hour (before the sea state changes) beach movement is limited ($R_1 < R_{\infty1}$) and the shoreline rotates only of an angle α_1 ($\alpha_1 < \beta_1$). As a result of the second sea state, wave conditions produced at the time t_2 generate

$R_{\infty 2}$ (dotted gray line) and $R(t_2)$ (gray line). Assuming l is the beach length (Fig. 3), R_∞ and R can be calculated geometrically as:

$$R_\infty = \frac{l}{2} \cdot \tan(\beta) \tag{5}$$

$$R(t) = \frac{l}{2} \cdot \tan(\alpha(t)). \tag{6}$$

Notice that we have dropped the time-dependency for variables defining equilibrium states and for variables describing the forcing conditions (including the depth of closure). Only for instantaneous values (i.e., for calculations within a defined sea-state), the time-dependency is maintained.

3.2.2. Characteristic time scale (T_s)

This term represents the time required for the beach to reach its equilibrium shape under certain forcing conditions. T_s increases with the length of the beach (more sediment needs to be moved to allow for plan-form rotation) and decreases for high energetic conditions (more energy is inputted into the system). In this section, an analytical expression for T_s will be developed. Similar to Kriebel (1986), and assuming that beaches have constant cross-shore profiles, the relative volume change is the same as the relative shoreline response

$$\frac{R(t)}{R_\infty} = \frac{V(t)}{V_\infty}. \tag{7}$$

Therefore, the volume change generated under steady forcing conditions can be described by the differential equation:

$$\frac{dV(t)}{dt} = \frac{1}{T_s} \cdot (V_\infty - V(t)). \tag{8}$$

According to Eq. (8), the rate of volumetric change, $\frac{dV}{dt}$, is proportional to the difference between the time-dependent volume (a steady forcing of 1 h is considered in the model), $V(t)$, and the equilibrium one, V_∞. These volumes can be determined geometrically as a function of R_∞ and $R(t)$:

$$V_\infty = 0.5 \cdot h^* \cdot \frac{l}{2} \cdot \tan(\beta) = 0.5 \cdot h^* \cdot \tan(\beta) \tag{9}$$

$$V(t) = 0.5 \cdot h^* \cdot \frac{l}{2} \cdot R(t) = 0.5 \cdot h^* \cdot \frac{l^2}{4} \cdot \tan(\alpha(t)) \tag{10}$$

where h^* is the closure depth of the beach profile (Fig. 5).

For the case of beach rotation, the rate of volume change is related to alongshore gradients in sediment transport. Inman and Bagnold (1963) and Komar and Inman (1970) described the alongshore volumetric sediment transport rate, S_l, as:

$$S_l = \frac{I_l}{(\rho_s - \rho_w) \cdot g \cdot a'} \tag{11}$$

where ρ_s and ρ_w represent the sediment and water density, respectively, g is gravity, a' a coefficient depending on the sediment porosity $P(a' = 1 - P)$ and I_l is the immersed-weight transport rate defined also by Inman and Bagnold (1963) as:

$$I_l = k \cdot EF_m \cdot \sin(\gamma_b(t)) \cdot \cos(\gamma_b(t)) \tag{12}$$

where $\gamma_b(t)$ represents the instantaneous angle between the wave crest at breaking and the long-term equilibrium shoreline, β. Referring back to Fig. 4, we can then define:

$$\gamma_b(t) = \beta - \alpha(t). \tag{13}$$

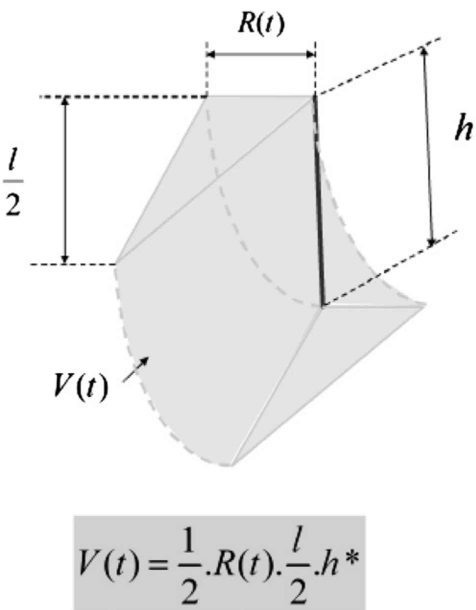

Fig. 5. Sketch of the simplified beach geometry considered for the evaluation of sand volumes associated to beach change between two successive time instances.

EF_m is the magnitude of the energy flux per unit of wave crest length. k represents a dimensionless proportionality coefficient which depends on the mean grain size of the sediment. Following Valle et al. (1993), this coefficient is defined as:

$$k = 1.4 \cdot e^{-2.5 \cdot D_{50}}. \tag{14}$$

According to hypothesis 5, alongshore sediment transport is parameterized only in terms of breaking wave quantities and plan-form beach rotation is assumed to be governed by currents associated to the oblique wave approach. Therefore, the rate of volumetric change $\left(\frac{dV(t)}{dt}\right)$ is described using Eqs. (11) and (12) as:

$$\frac{dV(t)}{dt} = K \cdot EF_m \cdot \sin(2 \cdot \gamma_b(t)) \tag{15}$$

where K is a coefficient defined as

$$K = \frac{k}{(\rho_s - \rho_w) \cdot g \cdot a'}. \tag{16}$$

Substituting Eqs. (9), (10) and (15) in Eq. (8), T_s can be expressed as:

$$T_s(t) = \frac{l^2 \cdot h^* \cdot (\tan\beta - \tan\alpha(t))}{4 \cdot \widehat{K} \cdot EF_m \cdot \sin(2 \cdot \gamma_b(t))} \tag{17}$$

where all variables have been previously defined. In this context it is worth reiterating that, within a defined sea state (each sea state is one-hour long) we assume steady wave conditions (β, EF_m and $h^*$are constant). EF_m is expressed as a function of the wave height (H_b), the wave celerity (C_b) at breaking and n_b (a coefficient relating the group celerity Cg to the celerity C):

$$EF_m = \frac{1}{8} \cdot \rho_w \cdot g \cdot {H_b}^2 \cdot C_b \cdot n_b. \tag{18}$$

Following Van Rijin et al. (2003), the initiation of sediment motion occurs above critical conditions which can be parameterized using the critical depth-averaged speed $\left(\hat{U}_{cr}\right)$:

$$U_{cr} = \left(0.014 \cdot T_P \cdot (s-1)^2 \cdot g^2 \cdot D_{50}\right)^{\frac{1}{3}}. \tag{19}$$

At the same time, the critical wave height H_{cr} is given as:

$$H_{cr} = \frac{1}{\pi} \cdot T_P \cdot \sin\left(\frac{2 \cdot \pi \cdot h}{L}\right) \cdot \hat{U}_{cr} \cdot \sqrt{2} \tag{20}$$

where T_s is the peak wave period, h is the water depth and L is the wavelength.

It follows that the magnitude of the energy flux required to move sediments, EF_r, can be written as

$$EF_r = \frac{1}{8} \cdot \rho \cdot g \cdot (H_b - H_{cr})^2 \cdot C_b \cdot n_b. \tag{21}$$

In line with other studies (e.g., Birkemeier, 1985) the closure depth has been expressed as a function of the local wave height using the expression by Capobianco et al. (1997):

$$h^* = C_c \cdot (H_s)^{0.67} \tag{22}$$

where C_c is a constant that differs from one beach to another and usually varies between 2.4 and 3.4 (we used a value of 2.8), and H_s is the significant wave height defined for each hourly sea state.

The terms $\alpha(t)$ and $\gamma_b(t)$ vary in time but a relation between both angles and the equilibrium angle can be defined using Fig. 4. At time t_1, the initial shoreline position (dashed–dotted black line) rotates by an angle $\alpha_1(t)$ and at the end of the beach moves a distance $R(t)$ (solid black line). The equilibrium terms are β_1 and $R_{\infty 1}$ (dotted black line). At time t_2 wave direction has not changed and, similar to changes produced at t_1 (same equilibrium quantities β and R_∞), the instantaneous $R_2(t)$ and $\alpha_2(t)$ are observed. In this case, the breaking angle $\gamma_{b2}(t)$ can be defined between the wave crest and the new shoreline position produced at t_1 (solid black line):

$$\gamma_{b_2}(t) = \beta - \alpha_1(t). \tag{23}$$

Under wave conditions produced at t_n the breaking angle $\gamma_{bn}(t)$ expressed as $\beta - \alpha_{n-1}(t)$.

The instantaneous beach angle $\alpha(t)$ can be expressed, using Eq. (6), as arctan $\left(\frac{2 \cdot R(t)}{l}\right)$ which can be approximated by $\left(\frac{2 \cdot R(t)}{l}\right)$ (α less than 15%). The term $\sin(2 \cdot \gamma_b(t))$ in Eq. (17) can be expressed as a function of β and α (Eq. (13)) resulting, after a few trigonometric transformations, into:

$$\sin(2\gamma_b(t)) = 2\sin(2 \cdot \beta) - \frac{4R(t)}{l} \cdot \cos(2 \cdot \beta). \tag{24}$$

Finally, using the previous simplifications, the characteristic time scale T_s is evaluated as:

$$T_s(t) = \frac{l^2 \cdot h^* \cdot (\tan\beta - \tan\alpha(t))}{4 \cdot \hat{K} \cdot EF_r \cdot \chi(\beta, R(t))} \tag{25}$$

where

$$\chi(\beta, R(t)) = \sin(2 \cdot \beta) - \frac{4 \cdot R(t)}{l} \cdot \cos(2 \cdot \beta). \tag{26}$$

The characteristic time scale $T_s(t)$ depends implicitly on the instantaneous shoreline response $R(t)$. This term is also proportional to the square beach length l^2 and the sediment grain size D_{50} and inversely proportional to the energy conditions EF_r. Therefore, T_s increases in beaches characterized by large l and coarse sediments (high D_{50}) and decreases under energetic conditions (high EF_r). Once T_s has been evaluated, the rate of shoreline response $\frac{dR}{dt}$ can be computed numerically using an explicit Euler method. More specifically, for a defined hourly sea-state, we use a 5-minute time step to solve Eqs. (25) and (3). Once converged (the solution for each 5-minute time step is iterative), the shoreline response $R(t)$ and the angle of beach rotation $\alpha(t)$ can be determined. In summary, we calculate:

1 $\gamma_b(t_i)$ as the angle equal to the difference between β and $\alpha(t_{i-1})$;
2 $T_s(t_i)$ and $R(t_i)$, iteratively, using the implicit form of Eqs. (24) and (3), respectively. At t_0 $(i = 0)$, the values of $T_s(t_0)$ and $R(t_0)$ are calculated from the previous sea state.
3 $\alpha(t_i)$ as the angle between the instantaneous shoreline position at t_i and the preceding one produced at t_{i-1}. At t_0 $(i = 0)$, the value of $\alpha(t_0)$ is the one obtained for the previous sea state.

Once converged, the model provides the response of the shoreline $R(t)$, associated to the sea state, and the angle of beach rotation $\alpha(t) = \arctan(2 * R(t) / l)$.

4. Results

4.1. Seasonal variability of shoreline response

Shoreline response, due to beach rotation, was simulated during a period of two year from April 2005 to April 2007 at Barcelona beaches. Because the 3 beaches analyzed are in close proximity, they experience the same wave climate and, more specifically, the same energy flux EF_r. As seen in Fig. 6, EF_r oscillates between 0 and 800 J/s with a mean value of 200 J/s. Low values are observed during summer (April–September 2005) and higher values are observed in winter (October 2005–March 2006).

Initial values of β and T_s were computed using the shoreline position extracted from the video images on 1-April-2005.

The equilibrium beach angle, β, and the characteristic time scale, T_s, calculated for one year are shown in Fig. 7. The equilibrium angle β oscillates around a mean value of zero and ranges approximately between $\pm 10°$ in Bogatell (mean of 1.5° and standard deviation of 4°), ± 12 in Nova Icaria (mean of 1.4° and standard deviation of 3.8°) and $\pm 15°$ in Somorrostro (mean of 1.1° and standard deviation of 3.5°). Positive/negative signs of β reflect the clockwise/counter-clockwise rotation of the beach plan-form and consequently the beach advance/retreat (Fig. 7a). With respect to the characteristic time scale, the mean values are 100 days at Somorrostro, 250 days at Nova Icaria and 500 days at

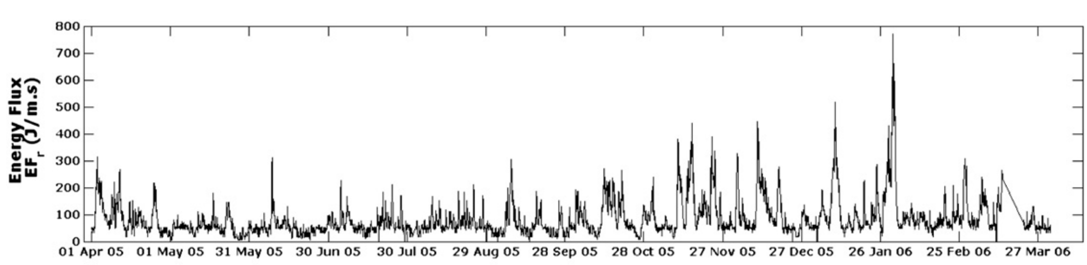

Fig. 6. Energy flux, EF_r, computed using Eq. (20), during April 2005–April 2006 at Barcelona beaches.

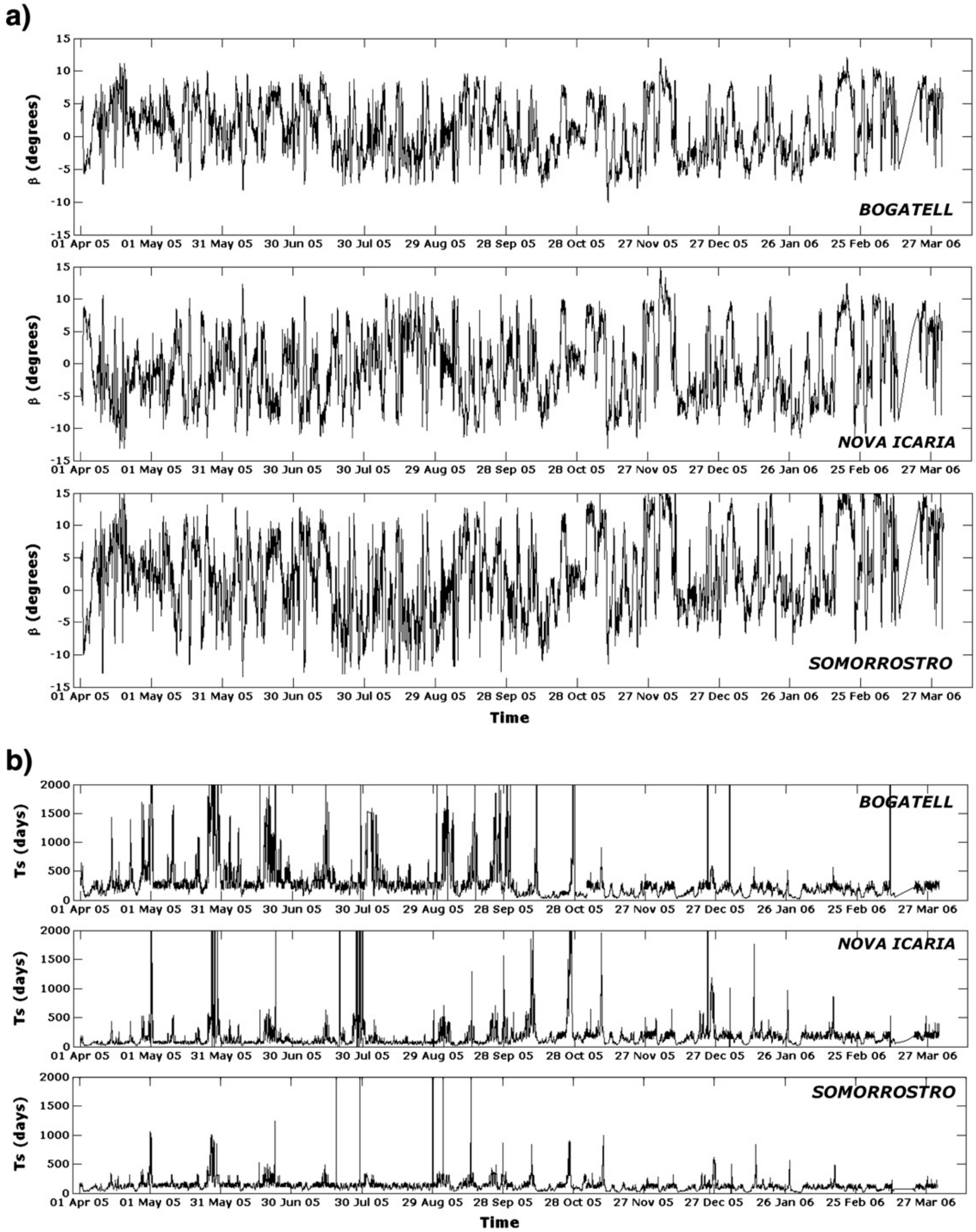

Fig. 7. Temporal variations in (a) β and (b) T_s during April 2005–April 2006 as computed by the numerical model.

Bogatell (Fig. 7b). Values of T_s increase during summer months when extended periods of low energy waves occur. Under low energy conditions, T_s is bigger than 1500 days for approximately 50 times at Bogatell, 20 times at Nova Icaria and 5 times at Somorrostro (the shorter beach with finer sediments). These values decrease with high-energy wave events in winter (between October 2005 and March 2006) when more than 80% of T_s is below the mean values for all beaches (Fig. 7b).

The evolution of the shoreline response, R, has been computed at the northern side (P_{10}) of each beach during April 2005–April 2006 using observed wave characteristics. The evolution of R (Fig. 8) displays a series of positive and negative changes in shoreline rotation. Seasonal variations can also be characterized by slow shoreline accretion in summer and fast erosion in winter.

Results obtained at the three beaches show that the instantaneous shoreline response is primarily controlled by the equilibrium beach angle and the characteristic time scale. The values of R are lowest in Bogatell, the longest beach ($l = 600$ m), than those detected in Nova Icaria which is a shorter beach with similar grain size ($D_{50} = 0.75$ mm), and increase even more in Somorrostro where the sediment grain size is finer ($D_{50} = 0.45$ mm). With respect to T_s, the highest number of days was observed in Bogatell and it becomes increasingly smaller in Nova Icaria and even more in Somorrostro.

As seen in Fig. 8, the initial shoreline position used for this simulation is 0.5 m at Bogatell (Fig. 8a), -4.5 m at Nova Icaria (Fig. 8b) and -2 m at Somorrostro (Fig. 8c). The model shows a good agreement with the observations with a root mean square error, RMSE, of 1 m at

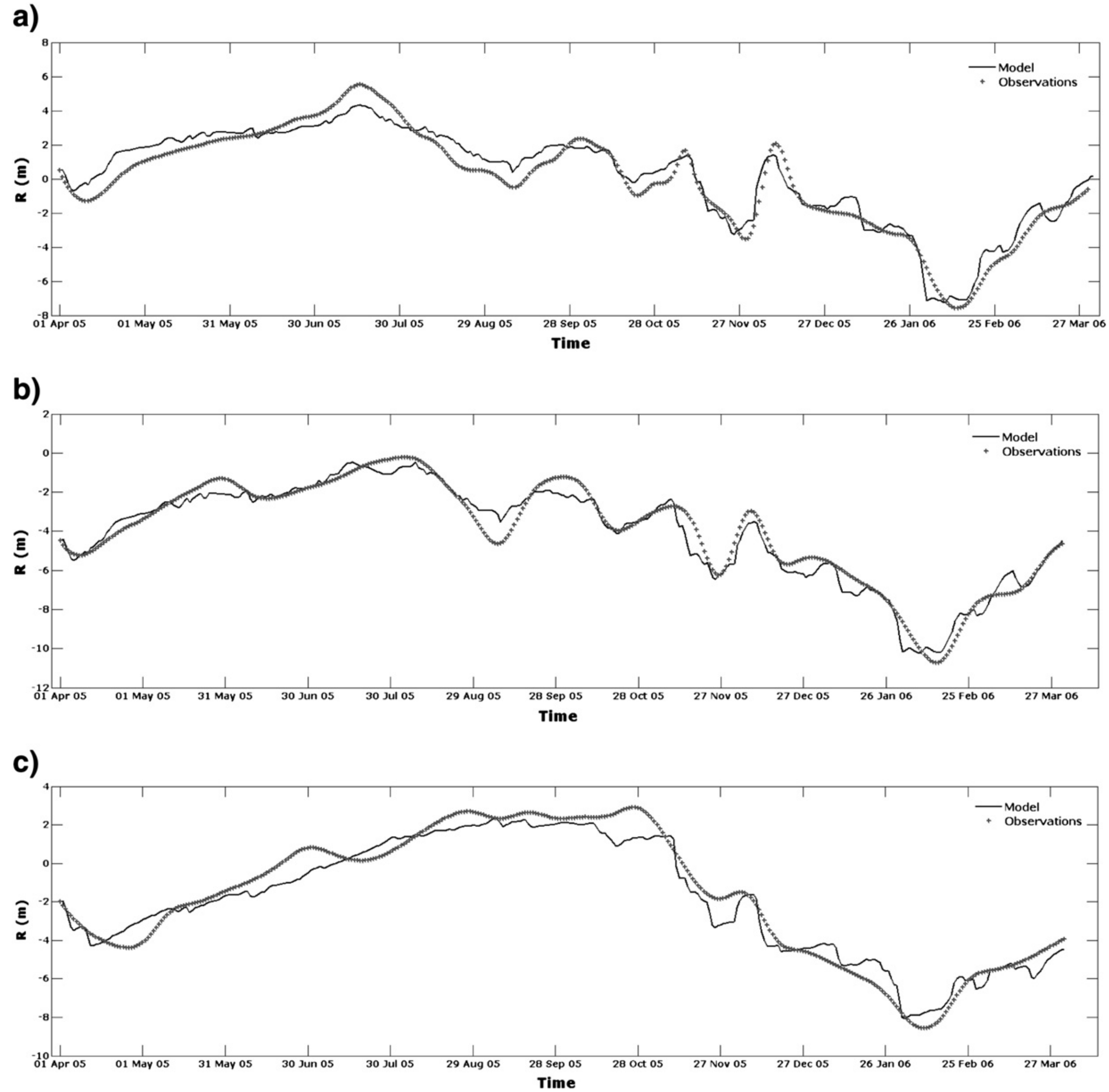

Fig. 8. Shoreline response *R* computed (continuous black line) during April 2005–April 2006 at Barcelona beaches: (a) Bogatell, (b) Nova Icaria and (c) Somorrostro. Field observations are also plotted (gray crosses).

Somorrostro, 1.16 m at Nova Icaria and 1.35 m at Bogatell. More importantly, the model seems capable of reproducing both the magnitude and temporal variations of accretion and erosive events. Observations actually seem to be smoother than actual observations and some of the small scale variability (e.g., end of 2005) is not well reproduced.

4.2. Inter-annual variability of shoreline response

The computation of the shoreline response over longer temporal scales was carried out for 17 years (1991 to 2008) using the hindcast wave data (Fig. 9a).

This computation was interested in Nova Icaria because it was the only beach that did not experience anthropogenic changes (e.g., beach nourishment) over the period of interest.

The mean value of H_b is 1.5 m and it reaches more than 4 m during high energy events such as those produced in December 1997, December 2001 and November 2003. Generally, waves come from ESE (110° with respect to the North) and SSE (160° with respect to the North).

The interannual shoreline response at Nova Icaria is shown in Fig. 9b. Since no video observations were available in 1991, the model simulation was started assigning a generic reference level ($R = 0$) as an initial condition for 1-Jan-1991. Observed shoreline position, extracted from video images between 2005 and 2007, were used to validate the numerical simulation which is in agreement with the observations (RMSE is 1.2 m while in Fig. 8 it was 1.16 m). Shoreline evolution appears to be controlled by seasonal variations with slow accretion corresponding to long periods of moderate wave action (summer time), and faster erosion under high energy conditions (winter time).

The modeled shoreline rotation appears to be controlled by a combination of specific wave conditions (H_s and Dir_b). For example, wave conditions in December 1997, are characterized by a high value of H_b (more than 5 m) but no significant change of wave direction which was between SE (135°) and SSE (160°) (Fig. 9a). These conditions resulted in small values of R and so of shoreline rotation. On the other hand, high energy is also observed in December 2001 when waves, coming from ESE direction (approximately 110° respect to the North reference), are

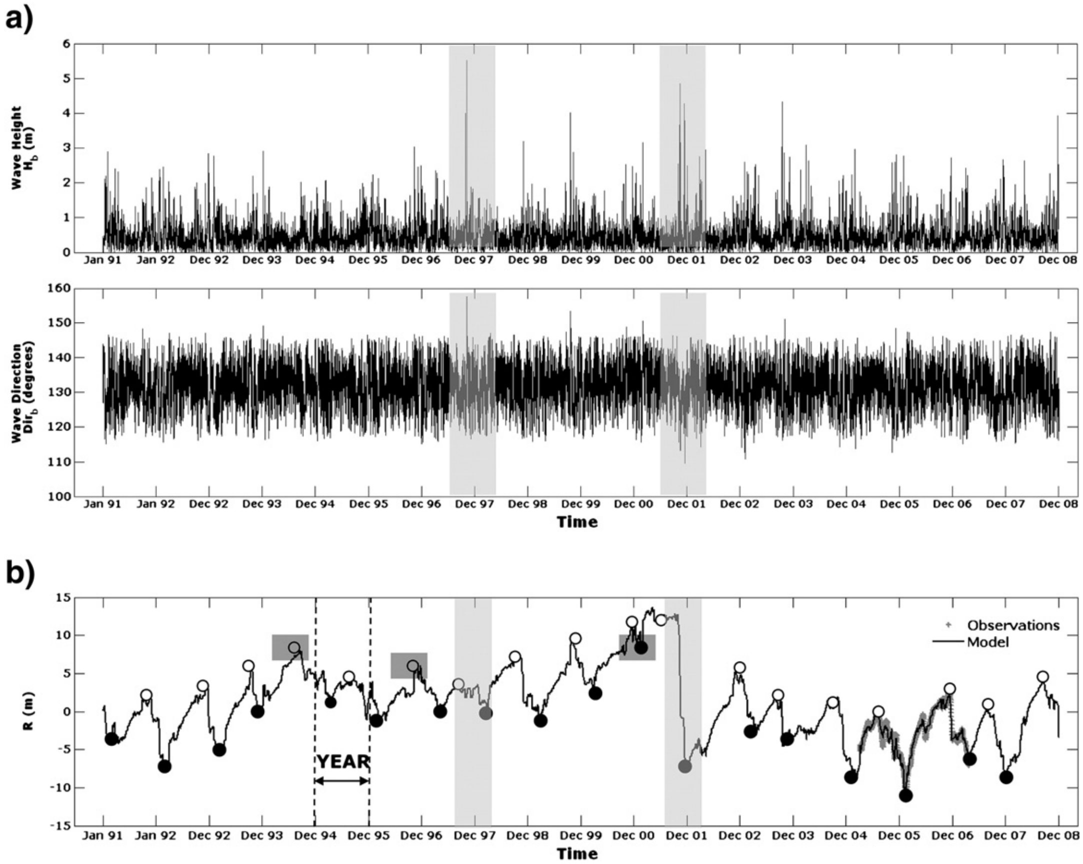

Fig. 9. (a) Time series of wave characteristics and (b) shoreline rotation at Nova Icaria beach over the period 1991 to 2008. R reproduces field observations of shoreline rotation extracted from video images between 2005 and 2007 (gray cross). White and black circles represent the maximum shoreline rotation in summer and winter, respectively.

more than 4 m. Under these conditions, R shows much larger negative changes of the order of 20 m (Fig. 9b).

5. Discussion

5.1. Summary of results

The model developed in this study uses the wave timeseries to reproduce the observed shoreline rotation at the northern end of the three embayed beaches of Barcelona. The model RMSE evaluated over two years ranges between 1 and 1.35 m. Modeling results for the 3 Barcelona beaches show that the magnitude of the plan-form change (quantified using the response variable R) is higher at shorter beaches with fine sediments, such as Somorrostro ($l = 400$ m and $D_{50} = 0.45$ mm), while the time required for the beach to respond to wave forcing (the characteristic time scale T_s) is smaller than that observed on beaches with larger length and coarser sediment such as Bogatell which is longer and coarser ($l = 600$ m and $D_{50} = 0.75$ mm). For all beaches, the characteristic time scale increases in summer when the wave action is moderate and the time needed for the shoreline rotation is bigger than that needed to move the plan-form during episodic storm events. The beach-evolution model is able to reproduce the interannual shoreline rotation at Nova Icaria beach as well as the seasonal responses in summer and winter. These responses show slow advance for long periods of low-energy conditions, and faster retreat during high-energy events. Shoreline change is more sensitive to variations in the incident wave direction than in wave height (Fig. 9).

5.2. Model assumptions and limitations

The model proposed here has the advantage of being deliberately simple, computationally efficient and with no calibration factor. Despite

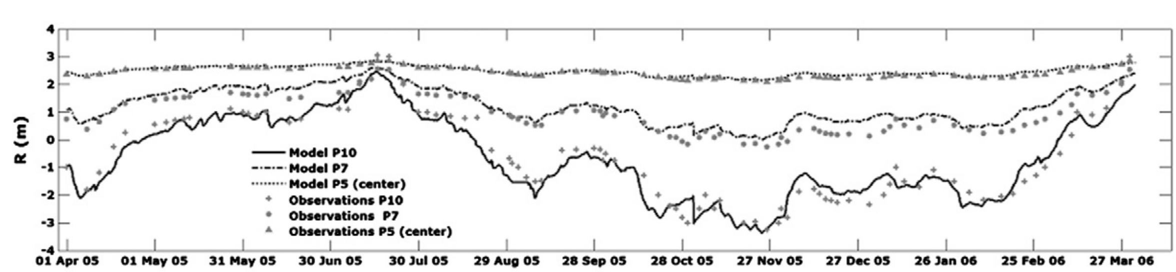

Fig. 10. Shoreline response at P_{10}, P_7 and P_5 at Somorrostro for the period April 2005–April 2006.

its simplicity, the numerical model is in good agreement with the observations. Nevertheless, it should be recognized that the present model is subjected to a number of hypotheses that might limit its applicability to other beaches. For example, the model neglects the feedback between changes in beach shape and the incident wave field. This assumption can be valid for short embayed beaches or limited beach rotation but it will result in increasingly larger errors (in terms of wave transformation, wave breaking location and longshore current generation) if the beach width or the angle of beach rotation increase. Assessing the role of these feedbacks will be the scope of future studies. Similarly, it is likely that the assumption of a linear shoreline rotating around a pivotal point is valid only for short beaches (as in this study). To test this hypothesis, we have computed shoreline change at other alongshore locations and compared them with the shoreline position acquired from video data (Fig. 10). Aside from the agreement between model and data, results show the presence of a fixed pivotal point and that shoreline changes occur approximately linearly between the different transects. Nevertheless, in more general terms, the assumption of a linear shoreline is stringent and likely to limit the applicability of this approach to short beaches.

5.3. Sensitivity to model parameters

Various model parameters can affect the performance of the model. In particular, the theoretical development of the characteristic time scale T_s, presented in Section 3.2.2, assumes that sediment is in motion once a critical wave height is exceeded. In the numerical model, this is taken into account when determining EF_r (Eq. (21)). Neglecting the critical wave height, the shoreline rotation could also be computed using EF_m (Eq. (18)) as shown in Fig. 11a (gray line). The difference between model outputs from the two cases (Fig. 11a, gray and black continuous lines) is primarily related to the initial period of low-energy waves associated with the summer of 2005 (see Figs. 2 and 9). During this period the model that evaluates T_s without considering a critical wave height overpredicts shoreline rotation. For this version of the model, sediment is always considered to be in motion even when the model that uses a wave height threshold predicts no sediment transport. Once wave height increases and large changes in the angle of wave approach are observed (see end of July 2005 in Fig. 2), the two model outputs converge and differences become negligible (Fig. 11a).

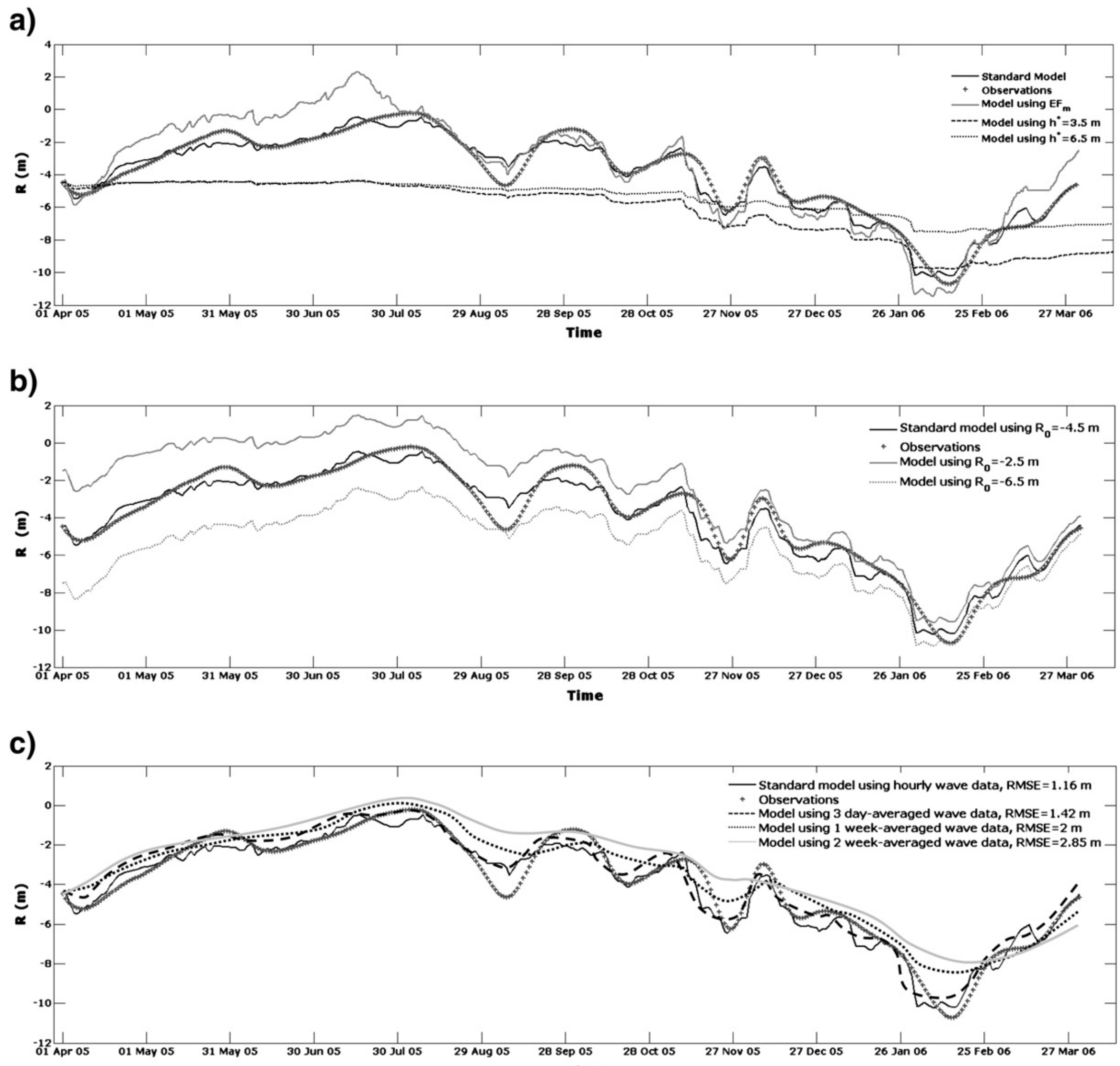

Fig. 11. Sensitivity of the beach-evolution model shown by the modeled shoreline rotation (solid-black line) at Nova Icaria beach over one year (2005–2006). The observed data are also plotted. (a) Computation of the shoreline rotation using (1) a constant closure depth and (2) EF_m for the energy flux. (b) Computation of the shoreline rotation using two initial shoreline positions R_0 different from the observed one ($R_0 = -4.5$ m): (1) $R_0 + 2$ m and (2) $R_0 - 2$ m. (c) R computed from wave data averaging over 3 (dashed-black line), 7 (dotted-black line) and 14 (solid-gray line) preceding days.

Evaluation of the depth of closure is still a topic of active research and model sensitivity to this parameter should be evaluated. Rather than using the parameterization shown in Eq. (22) (Capobianco et al., 1997), shoreline rotation has been computed using constant values of the closure depth (3.5 m and 6.5 m, dashed and dotted black lines in Fig. 11a, respectively). Results show that this parameter critically controls the modeled shoreline response. Compared to results obtained with the standard model, using a high value of constant closure depth (6.5 m) implies a larger characteristic time scale which in turn implies limited changes in the shoreline response, especially during summer time (May 2005–September 2005) when wave action is moderate. A smaller constant value of the closure depth (3.5 m) results in smaller values of T_s which can at least qualitatively reproduce some of the behavior observed in winter months (November 2005–March 2006) when the shoreline response decreases. Overall, a non constant closure depth (varying with time and depending on wave action) and critical energy conditions seems to be a key requirement for the beach-evolution model to successfully reproduce shoreline rotation.

5.4. Sensitivity to initial conditions and averaging of wave forcing

Shoreline rotation was modeled using the observed shoreline position (1-April-2005) as initial condition. Here, the sensitivity of the model to the initial value of R was analyzed using different values of the initial shoreline position (-1.5 m and -7.5 m). Fig. 11b shows that the three curves display the same type of response to the wave climate and the predicted shoreline response converges over time so that after about 8 months the three shoreline responses are essentially indistinguishable. Determination of the exact amount of time required for model outputs to converge will be the topic of future studies.

Hourly wave parameters (H_b, Dir_b, and T_P) were used to compute R and β but temporal averages of wave data (from days to weeks) could also be used to simplify the numerical computations or in cases when hourly measurements are not available. However, temporal averaging of the wave data reduces the model performance which, for averages longer than a week, fails to resolve the shoreline response. Fig. 11c shows model results for a period of one year (from April 2005 to April 2006) using different wave averages over preceding days. As observed in Fig. 11c, R, computed using wave energy values averaged over the previous three-days, is roughly similar to the one obtained by the hourly wave energy. Longer averaging of the incoming wave energy results in a smoothed evolution of the shoreline because (a) extremes in wave characteristics are smoothed out through averaging and (b) timing of extreme events is diffused over the temporal scale of wave averaging.

5.5. Sensitivity to decorrelation between cross-shore and longshore processes

The beach-evolution model was developed under the assumption that changes in the alongshore and cross-shore direction are independent and can be analyzed separately (which is the case for the beaches analyzed in this study, see also Turki et al., 2013). This separation is not a universal feature of pocket/embayed beaches and observations have been reported (e.g., Harley et al., 2012) where longshore and cross-shore movements are mutually interdependent and cannot be separated. Although no test has been performed, it is likely that in these cases the skills of the beach-evolution model will decay because some of the hypotheses over which the model is built are likely to be invalid (e.g., linear shoreline, uniform cross-shore profiles, and lack of gradients in wave height). Furthermore, the model assumes that the beach volume is generally constant and no 'net loss' or 'net gain' of sand is observed. Under these assumptions, changes in shoreline position can be mainly attributed to alongshore transport of sediment from one side of the beach to the other. This assumption is generally fulfilled only on short or artificial embayed beaches, where the presence of headlands or the use of groins prevents sediment losses.

The numerical model was developed assuming a constant wave height along the beach. Consequently, we have also assumed a linear beach response to wave action and considered the cross-shore translation of the shoreline of one end of the beach (the northern side, P_{10}, in the present study) as a representative proxy of shoreline rotation. These conditions are generally fulfilled only for short beaches where spatial variations in wave height are negligible and the detailed structure of the nearshore hydrodynamic circulation can be ignored. Beaches with parabolic plan-forms are mainly generated by gradients in wave height which are in turn responsible for gradients in longshore sediment transport. The shoreline response along these beaches cannot be resolved by the present version of the model.

5.6. Future work

The present version of the model requires the use of predictive expressions for the total alongshore transport rate. Other sediment transport formulae could be considered to check model sensitivity and possibly to include the effect of alongshore gradients in wave height (e.g., Ozasa and Brampton, 1980). This will help extending the present model and relax some of the more stringent hypotheses used to predict shoreline rotation.

Finally, since the model was intended to simulate three beaches on the Mediterranean sea, little attention has been paid to the tidal range. Tides at Barcelona beaches are small (range is about 0.2 m) and the governing equations assume that wave action producing longshore sand transport and offshore wave conditions are the major factors controlling the shoreline response. For macrotidal beaches, the presence of tides would affect the amount of time the upper part of the beach is active in a morphodynamic sense. Also, the tidal range could be taken into account and affect, for example, the evaluation of the depth of closure (Eq. (21)). More field testing under a variety of conditions is needed to fully assess the role of tides and the possibility of applying the model to longer macrotidal embayed beaches.

6. Conclusions

A new beach evolution model relating the instantaneous change of shoreline rotation to the equilibrium plan-form of the shoreline was developed. The model is able to reproduce the plan-form response of 3 pocket beaches using as an input the wave timeseries and the physical characteristics of the beach. Governing equations were simplified assuming that the gradients in alongshore sediment transport are negligible and that the beach plan-form is linear. The model has been applied to three pocket beaches of Barcelona (Spain) and is able to successfully predict the shoreline position. Results from the numerical model were compared with shoreline positions extracted from video images and, over a period of two years, the RMSE is less than 1.5 m. The key advantage of our model is that it is simple (no calibration factor is needed) and efficiently predicts long-term shoreline rotation. Overall, this model provides a new powerful tool to advance understanding of physical processes of the dynamics of pocket beaches and to predict its future evolution.

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References

Birkemeier, W.A., 1985. Field data on seaward limit of profile change. Journal of Waterway, Port, Coastal, and Ocean Engineering 111, 569–587.

Boker-Hedegaad, I., Deigaard, R., 1991. Onshore/offshore sediment transport and morphological modelling of coastal profiles, in processing. Coastal Sediments, 91 643–657.

Capobianco, M., Larson, M., Nicholls, R.J., Kraus, N.C., 1997. Depth of Closure: A Contribution of the Reconciliation of Theory, Practise and Evidence, in Coastal Dynamic, 97, pp. 506–515.

Dabees, M., Kamphius, J.W., 2000. N-line: efficient modelling of 3-d beach change. In: New York ASCE (Ed.), Proceeding. 27th Coastal Engineering Conference. ASCE, pp. 2700–2713.

Davidson, M.A., Turner, I.L., 2008. A behavioural-template beach profile model for predicting seasonal to interannual shoreline evolution. Journal of Geophysical Research — Earth Surface 24, 114. http://dx.doi.org/10.1029/2007JF000888.

Davidson, M., Turner, I.L., Guza, R.T., 2011. The effect of temporal wave averaging on the performance of an empirical shoreline evolution model. Coastal Engineering 58, 802–805.

De Vriend, H.J.D., Capobianco, M., Chesher, T., Swart, T., 1993a. Approaches to long-term modelling of coastal morphology. Coastal Engineering 21, 225–269.

De Vriend, H.J.D., Zyserman, J., Roelvink, J., 1993b. Medium-term 2DH coastal area modelling. Coastal Engineering 21, 193–224.

Dean, R.G., 1973. Heuristic models of sand transport in the surf zone. Paper presented at Conference on Engineering Dynamics in the Surf Zone. Institute of Engineering, Sydney, N.S.W., Australia.

Dean, R.G., 1977. Equilibrium Beach Profiles: U.S. Atlantic and Gulf Coasts. Technical Report.

Gourlay, M.R., 1968. Beach and Dune Erosion Tests. Delft Hydraulics Laboratory, Delft, Netherlands (Rep. m935/m936).

Hanson, H., 1989. GENESIS: a generalised shoreline change numerical model. Journal of Coastal Research 5, 1–27.

Hanson, H., Kraus, N.C., 1989. Genesis: generalized model for simulating shoreline change. Proceeding. 30th International Conference on Coastal Engineering, 4, pp. 3762–3773.

Harley, M.D., Turner, I.L., Short, A.D., Ranasinghe, R., 2010. Interannual variability and controls of the Sydney wave climate. International Journal of Climatology 30, 1322–1335.

Harley, M.D., Turner, I.L., Short, A.D., Ranasinghe, R., 2012. A re-evaluation of coastal embayment rotation: the dominance of cross-shore versus alongshore sediment transport processes, Collaroy–Narrabeen, beach, Australia. Journal of Geophysical Research 116, F04033. http://dx.doi.org/10.1029/2011JF001,989.

Holman, R.A., Stanley, J., 2007. The history and technical capabilities of Argus. Coastal Engineering 54, 477–491.

Inman, D.L., Bagnold, R.A., 1963. Littoral Processes. , vol. 3. Wiley-Interscience, New York.

Komar, P.D., 1998. Beach Processes and Sedimentation. (Ph.D. thesis) Prentice Hall, Upper Saddle River (N.J.).

Komar, P.D., Inman, D.L., 1970. Longshore sand transport on beaches. Journal of Geophysical Research 75 (30), 5914–5927.

Kraus, N.C., 1991. Generalized Model for Simulating Shoreline Change. Technical Report.Department of Water Resources Engineering Lund Institute of Science and Technology, University of Lund, Box 118, Lund, Sweden.

Kriebel, D.L., 1986. Verification study of a dune erosion model. Shore and Beach, 54 (3) 13–21.

Kriebel, D., 1998. Beach and Dune Response to Hurricanes. (Ph.D. thesis) Department of Civil Engineering, University of Delaware, Newark, DE.

Kriebel, D., Dean, R.G., 1970. Numerical simulation of time-dependent beach and dune erosion. Coastal Engineering 9, 221–245.

Kriebel, L.D., Dean, R.G., 1985. Numerical simulation of time-dependent beach and dune erosion. Journal of Waterway, Port, Coastal, and Ocean Engineering 9, 221–245.

Larson, M., Kraus, N.C., 1989. Sbeach: Numerical Model to Simulate Storm-induced Beach Change. Technical Report. U.S. Army Corps of Engineers, Waterway Experiment Station, Vicksburg.

Miller, J., Dean, R., 2004. A simple new shoreline change model. Coastal Engineering 51 (7), 531–556.

Miller, J.K., Dean, R., 2007. Part II Shoreline variability via empirical orthogonal function analysis: relationship to nearshore conditions. Coastal Engineering 54, 133–150.

Ojeda, E., Guillén, J., 2008. Shoreline dynamics of embayed beaches. Marine Geology 253, 51–62. http://dx.doi.org/10.2112/07-0886.1.

Ozasa, H., Brampton, A.H., 1980. Mathematical modelling of beaches backed by seawalls. Coastal Engineering 4 (1), 47–63.

Pernard-Considère, R., 1954. Essai de théorie de l'évolution des formes de rivage en plages de sable et de galets, in IV Journées de l'Hydraulique, Les énergies de la Mer, Paris.

Plant, N.G., Holman, R.A., Freilich, M.H., Birkemeier, W.A., 1999. A simple model for interannual bar behaviour. Journal of Geophysical Research 104 (C7), 15,755–15,776.

Reguero, B.G., Mendez, F.J., Minguez, R., Losada, I.J., 2012. A global ocean wave (GOW) calibrated reanalysis from 1948 onwards. Coastal Engineering. http://dx.doi.org/10.1016/j.coastaleng.2012.03.003.

Roelvink, J.A., Broker, I., 1993. Cross-shore profile models. Coastal Engineering 21, 163–191.

Roelvink, J.A., Van Banning, G.K.F.M., 1994. Design and development of DELFT3D and application to coastal morphodynamics. Hydroinformatics, 94 451–455.

Short, A.D., Trembanis, A.C., Turner, I.L., 2000. Beach oscillation, rotation and the southern oscillation, Narraben beach. Proceeding. 27th International Conference on Coastal Engineering, Australia, pp. 2439–2456.

Stive, M.J.F., Battjes, J.A., 1984. A model for offshore sediment transport. Proceeding. 19th Coastal Engineering Conference. ASCE, New York, pp. 1420–1436.

Sunamura, T., Maruyama, K., 1987. Wave-induced geomorphic response of eroding beaches with special reference to seaward migrating bars. Proceedings of Coastal Sediments 87, New Orleans, pp. 788–801.

Swart, D.H., 1974. Offshore Sediment Transport and Equilibrium Beach Profiles. (Ph.D. thesis) Delft Hydraulics Laboratory, The Netherlands.

Turki, I., 2011. Department of Science and Water and Environmental Techniques. (Ph.D. thesis) University of Cantabria, Santander, Spain.

Turki, I., Medina, R., Gonzalez, M., Coco, G., 2013. Natural variability of shoreline position: observations at three pocket beaches. Marine Geology 338, 76–89. http://dx.doi.org/10.1016/j.margeo.2012.10.007.

Valle, D., Medina, R., Losada, M.A., 1993. Dependence of coefficient k on grain size. Journal of Waterway, Port, Coastal, and Ocean Engineering 568–574.

Van Rijin, A.G., Walstra, D.J.R., Sutherland, B., 2003. The predictability of cross-shore bed evolution of sandy beaches at the time scale of storms and seasons using process-based profile models. Coastal Engineering 47, 295–327.

Werner, B.T., 2003. Modeling landforms as self-organized, hierarchical dynamical systems. In: Wilcock, P.R., Iverson, R.M. (Eds.), Prediction in Geomorphology. Geophysical Monograph Series. American Geophysical Union, Washington, DC, USA, pp. 133–150.

Wright, L., Short, A., Green, M., 1985. Short-term changes in the morphodynamic states of beaches and surf zones: an empirical predictive model. Marine Geology 62, 339–364.

Yates, M.L., Guza, R.T., Reilly, W.C.O., 2009. Equilibrium shoreline response: observations and modelling. Journal of Geophysical Research 114. http://dx.doi.org/10.1029/2009JC005359.

Zheng, I., Dean, R.G., 1997. Numerical models and intercomparisons of beach profile evolution. Coastal Engineering 30, 169–201.